

Notes on K-12 Draft for MN received about 11 March 2003

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I have been through the draft more carefully than I had earlier expected, and you will find a lot of negative comments below, mostly with suggestions for improvement or restatement. I liked the brisk way it was written but found its pacing of subject matter less complete than I at first thought, or at any rate somewhat behind what I consider a reasonable schedule. But please don't take the fact that all the comments below are negative as meaning the document doesn't have the possibility of being good; it's just that I didn't waste time praising the good parts.

So here is my commentary:

In general the speed in K-5 is too slow, and requires a difficult speed-up in Grades 6, 7 and 8 to get to algebra. This can be remedied by removing some of the "Data Analysis" in the early grades and putting instead some more arithmetic, including explicit demands for the paper-and-pencil algorithms for decimal and fraction arithmetic, as well as more attention to specific examples of "story problems". These latter are sometimes implied in the Mathematical Reasoning sections, but vaguely.

In particular, Grade 6 Fractions suddenly, in 6.4, page 31 bottom, asks for long division of multi-digit integers, and on page 32 for decimals (which includes integers, after all), and on page 32 the same for fractions, and indeed these things belong in Grade 6, but at the top of page 31, under Number sense, 6.3 sounds like something out of Grade 4, and is surely far behind the computations asked for under Computation. But Grade 4 includes nothing under Fractions, and Grade 5 has only 5.1 on p.27, which is fairly vague except for its specification of "simple" problems.

Equivalent fractions should be introduced and used freely by Grade 4, and Grade 5 should already be adding and multiplying any two fractions, making use of common denominators, so that Grade 6 can deal with division of fractions and its meaning.

I believe this slowness in K-5 is the source of so many of the "Public comments" on your web page, complaining that certain things in the middle school are "too hard" or "developmentally inappropriate". They aren't, but they have been inadequately prepared for in the present draft's earlier grades. Fraction and decimal arithmetic are the major job of K-6, with associated word problems of increasing complexity as the grades go on. The data analysis in K-3 is totally

unnecessary, though one should know about the number line and how to express numbers, so that perhaps a bar graph can be permitted as a relief from plain arithmetic and naming geometric figures.

Then Grade 7 can introduce symbols for numbers, to exhibit such things as evaluating formulas (area of circle, say, or radius of circle if area is given) and solving proportions, and full algebraic notation and its use in story problems and plotting functions in Grade 8.

All this is fairly general, but I cannot rewrite the whole thing here, and I have not studied the document in all detail. Here is a list (not a complete critique, however) of comments I have put in red on my copy of the draft. The document uses identifications such as "6.1" many times in Grade 6, according to the heading (Geometry, Number Sense, etc.). This makes it hard for me to give references, but I will give page numbers too.

Page 3: Mathematical Reasoning asks for use of 'technology' under Strategies. I would not prescribe any technology except alphabet blocks and M & M candies until Grade 5 or so. And Graphs in K.2 are quite pointless and time-wasting in Kindergarten. Indeed, I think Kindergarten children are likely to misunderstand the whole idea of a graph at that stage. First, they need to understand how number is correlated with lengths, after all.

Page 7: Top 1.1 should read "... a set of objects..."

Page 8: Bottom, 1.8.7 b asks for "solve problems..." Here and in many other places this phrase is used but not elucidated. I believe the writer thinks it obvious what sort of problem should occur at each such stage, but it is not. The Standards must give a clue, if only to writers of textbooks, what level problem is intended.

Page 9: Probability. I see no reason to pay any attention at all to probability. What is asked for here is trivial, and can be done in Grade 3 or 4 in five minutes, but will consume valuable arithmetic and measurement time here.

Page 10: Graphs. Same comment as above.

Page 11: Equivalence 2.1.12 mentions some sort of manipulative I never heard of: "10 rod, full tens frame". Standards should be about mathematics, not about the devices used in particular programs. This item should state what it is about number that the program wants, not what it wants about such devices one might use to teach that.

Page 13: Bottom, 2.7.2 is incoherently printed. Also, the vertices of a solid (or plane, for that matter) object should not be called corners.

Page 14: Data. I would save all this for Grade 3 at the earliest, but that is perhaps idiosyncratic. Still, if you read 2.1 you will see that the stuff in Grade 1 under Data is not needed if one were to want to begin Data here. I believe that if you look at page 19 under Data and Probability you will find that nothing in the earlier curriculum under Data was in fact needed to begin the whole subject here in Grade 3. So, why waste that earlier time?

Page 16: Does not say, under "computation" whether calculators are to be used. It should explicitly state their non-use. I see that equivalent fractions are introduced here; good. Grade 4 should repeat this, though at a deeper level, to include *all* denominators, and with explicit reference to the ordering on the number line.

Page 18: Under algebraic thinking: Letters can be used to represent known numbers, too, and not just pi. Children should get used to letters when general statements are made about all numbers. For example, for every integer n , $2n$ is an even number. Here "n" is used for a number, but there is nothing particularly unknown about the number or numbers we have in mind.

Page 19: Top 3.5 asks for elapsed time in quarter-hours; why not by the minute? After all, two-digit subtraction problems are included in Grade 3. Under Probability, too much time is allotted, or seems to be allotted to trivialities that should serve as a fast preamble to some real problems in later grades, say Grade 8. It turns up again and again and again and again, grade after grade; why?

Page 20: Like the Reasoning strand in other grades, more or less the same collection of words. It would be well to specify the depth of reasoning wanted in each grade, or not repeat these things. If you look at the Japanese Standards you will see how subtly they convey the increase of sophistication as the child gets older.

Page 22: Should not addition of simple fractions be introduced in Grade 4? The section under "fractions" in the middle of the page should not be empty.

Page 23: Top 4.1 "Demonstrate understanding of" should be replaced by some definite instruction, such as the sort of problem one should be able to solve using the property (which is in fact done here), or the phrase, "The student will use the distributive law in whole number computations." The section 3 Algebraic Thinking begins "The student will..." and then turns syntactically illogical in 4.2, where a new sentence begins. 4.3 is incoherent at the end.

Page 25: Is repetitious as mentioned earlier in connection with the recurring Reasoning strand. Try to indicate where reasoning is important in Grade 4 where it could not have been asked for in earlier grades, or else remain silent.

Page 26: Equivalence asks for decimal equivalence of common fractions, but it does not appear that a method of getting them (by decimal division) is yet available to students at this level. Only integer division with remainder seems to be required here, in 5.4 and under Computation on page 27.

Page 27: Fractions should contain much more here about equivalent fractions, arithmetic of fractions and interpretations, for fractions with (in principle) any denominators at all. This was mentioned above in my general remarks.

Page 28: It should be possible to ask in 5.2 for a proof of the fact that the angles of a triangle add to 180, but using some obvious properties of parallels and transversals, which are part of 5.1. Of course all this should be revisited in later grades.

Page 29: Statistics. A waste of time in Grade 5, though "mean" should be known as an item of vocabulary and an exercise in arithmetic.

Page 30: If the angles of a triangle add to 180, one can prove with an auxiliary line that the angles of a quadrilateral add to 360; why not put this here as an example of Reasoning available to Grade 5?

Page 31: Bottom: 6.3 and 6.4 say "without a calculator", a phrase that should occur in some of the earlier grades as well; otherwise the document is ambiguous or incomplete, or inconsistent. Someone should go through the whole thing for consistency before it is final.

Page 32: under Ratio etc: 6.3 should say "... when a percent of it is known." The syntax other wise permits the two uses of the word "number" to refer to two different numbers.

Page 33: 6.6 Omit all reference to the "concept" of "variable". It is not helpful, even though the word can be used casually as in 6.11 and 6.12 below. Trying to make sense of variable as a "concept" gives rise to many misunderstandings. It is simply not needed. It is a word, with definite meaning in certain contexts, but some people think there is such a thing as a variable number. There is not. All numbers I have ever met stand still while I look at them. Best leave the matter lie. But 6.11 and 6.12 are perfectly good just the same. When evaluating expressions containing letters representing numbers not named for the time being, we call those letters variables; that's all. No "concept" at all at all.

Page 34: Top, are the constructions to be made with straight-edge and compass? If so, say so. However, I think this is a bit too much for Grade 6. It takes time, and should be spread out a bit more.

Page 36: Here one can ask, for example, that the student be able to give arguments to show that the constructions asked for on page 34 really do the announced job.

Page 37: The first 7.1 invokes both positive and negative powers of 10 but the very next 7.1 limits the instruction to positive powers only. Why? I would remove the restriction. Under Fractions, 7.2 asks for the knowledge that b/a is the reciprocal of a/b . It seems rather late for this, which is one of the things I was urging above when I complained that not enough was done with fractions in Grades 4-6.

Bottom of page: I would omit all mention of irrational numbers until a later grade, probably Grade 8. That decimal approximations to the square root of 2, for example, can never be made exact in a finite number of digits is not a very

curious fact, and need not imply the existence of irrational numbers. $1/3$ cannot be expressed exactly in a finite number of decimal digits, either.

Page 38: Bottom: 7.1 invokes only ordered pairs of integers to be plotted on the plane, yet 7.3 wants the graph of a linear function, which needs more than integral points. The restriction in 7.1 should be removed.

Page 40: Everything in Data and Probability in Grades 1-6 can be included here, plus what's here, without strain, and with much gain of time in earlier grades.

Page 42: Computation, Irrational Numbers, computing with irrational numbers means necessarily learning to use approximations, and should be rewritten accordingly. Otherwise the computation is trivial: The only way I know to add π to $\sqrt{2}$ is to write " $\pi + \sqrt{2}$ ". So Item 8.1 needs omission or restatement. On the other hand, Grade 8 is a nice place to *introduce* irrational numbers and assert their existence. Proving things about them is probably beyond the scope of Grade 8, though if there is sufficient previous experience with reasoning one can prove the irrationality of $\sqrt{2}$, and if there is a good understanding of the long division algorithm one can prove that rational numbers have repeating decimal expansions. These things can be put under Reasoning at the end of the Grade 8 Standards.

Page 43: Under Algebraic thinking: I don't understand 8.5 at all. 8.10 speaks of "equations", but I don't see where the example illustrates the use of that word. 8.11 is badly stated, but I won't restate it here. In 8.13, if the criterion for perpendicularity is given (product of slopes), it should be *proved* using the Pythagorean theorem and the definition of slope. This is for Reasoning, too.

Page 45: 8.7 is an example of an erroneous mathematical usage that is unfortunately popular these days in ignorant books. Mathematical statements are either true or false, and they don't change from one day to another. In other words, there are no statements that are "sometimes" true, on Tuesday and not Wednesday, say. What is meant here is that a statement containing an unspecified number, represented by a letter, may be true for all numbers that replace the letter, some numbers that replace the letter, or no number that can replace the letter. Thus, we don't say " $x+y=3$ is sometimes true", but rather say "For some, but not all, values of x and y , $x+y=3$." I would rephrase 8.7 or omit it. Omitting it is wiser, since what it represents turns up in any problem involving solving equations or proving identities.